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THE BLAZHKO EFFECT OF AR HERCULIS

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ABSTRACT

V-band observations of the RR Lyrae star AR Her were obtained in 1992 and 1994–1995. During these years, AR Her continued to exhibit a strong Blazhko effect, showing light-curve variations more complex than pure amplitude modulation. The light curves of AR Her in 1992 and 1995 were modeled and compared with prior results. The light curve of AR Her is not a simple superposition of the light curves of an RR*ab* and an RR*c* star. AR Her is therefore not, as has been suggested, a composite system made of an RR*ab* and an RR*c* component. Coincident changes in primary and Blazhko period have been reported for some Blazhko variables. These period changes may possibly occur in AR Her, but stronger evidence for them has been seen in other stars. These changes suggest that the Blazhko period is not always equal to the rotation period of a star showing the Blazhko effect, in contradiction to recent rotator-pulsator models.

Key words: RR Lyrae variable — stars: magnetic fields

1. INTRODUCTION

The Blazhko effect is a periodic modulation of the shape and amplitude of the light curve of an RR Lyrae star on a timescale that may be as short as 10.9 days or as long as several hundred days (Smith 1995; Szeidl 1988; Teays 1993). The classical Blazhko effect occurs mainly and perhaps wholly among RR Lyrae stars of type RR*ab* (i.e., those that pulsate mainly in the fundamental radial mode) and very rarely, if at all, among RR Lyrae variables of type RR*c* (which pulsate mainly in the first-overtone radial mode). Perhaps 20% or more of the RR*ab* variables exhibit the Blazhko effect. Some nine decades after the discovery of the Blazhko effect, there exists no theory explaining it that commands general assent.

Smith et al. (1994) discussed photometric observations of AH Cam, the RR Lyrae star with the shortest known Blazhko period. In this paper we present and analyze new photometric observations of a second Blazhko star, AR Herculis. It has been known since the 1930s that AR Her exhibits the Blazhko effect, with a Blazhko period of about 31.6 days (Balazs & Detre 1939). In several well-studied RR Lyrae stars, the amplitude of the Blazhko effect has been observed to be variable on a timescale of years, in some cases in a quasi-periodic fashion. RR Lyrae itself is noteworthy in this respect, since its 41 day Blazhko cycle has almost disappeared in certain years (Detre & Szeidl 1973). One motivation of the present study is thus to extend the observational record of AR Her so as to compare its past and recent behavior.

There is a second reason for studying AR Her. It may be an unusually hot RR*ab* star. Its hydrogen lines at minimum light were found to be unusually strong for an RR*ab* star (Preston 1959). Moreover, Sturch (1966) found that its color at minimum is bluer than usual in $B-V$. Although neither Preston nor Sturch observed AR Her through a complete Blazhko cycle, they did not notice similar anomalies among other Blazhko stars in their samples. Kinman & Caretta (1992) suggested that AR Her might in fact be a binary RR Lyrae star, composed of an RR*ab* star with a 0.47 day period and an RR*c* star with a period of 0.23 days. Under their hypothesis, the binary nature of AR Her would explain both its bluer color and modulated light curve. Although we shall argue against the binary RR Lyrae interpretation, it may be that AR Her is very near, or indeed beyond, the usual blue edge to the RR*ab* instability strip.

2. OBSERVATIONS

CCD observations of AR Her in the V band were begun in 1992 at the 60 cm telescope of Michigan State University. The instrumental setup for these observations is identical to that used for the 1991–1992 observations of AH Cam, as described in Smith et al. (1994). No observations were obtained in 1993, but observations were resumed in 1994 and (mainly) 1995 using an SBIG ST-6 CCD. Exposure times were 30 s or 1 minute. In total, 1147 observations were obtained.

Aperture photometry of AR Her and two comparison stars (*HST* Guide Star Catalog stars 3491-134 and 3491-124) were obtained for each frame. The Guide Star Catalog V-band magnitudes for the two comparison stars are 12.25 and 13.79, respectively. Since these values are uncertain by about 0.3 mag, the photometry of AR Her is presented differentially with respect to the V magnitude of 3491-134. Because two CCD detectors were used and observations were made only in V, the question arises whether the 1992 and 1994–1995 magnitudes are on the same photometric system. Comparisons of various standard stars of different color observed with the two CCD cameras indicate that a small systematic difference, of order ± 0.02 mag, cannot be excluded. However, such a small difference is unimportant for the analyses carried out here. The uncertainty of a typical observation in 1992 is ± 0.015 mag in ΔV. The uncertainty of a typical observation in 1994–1995 is ± 0.02 mag. Observations are listed in Table 1.

The light curve of the entire set of observations is shown in Figure 1. Observations obtained in 1992 and 1995, the 2 years in which most of the observations were obtained, are

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TABLE 1

PHOTOMETRY OF AR HERCULIS

HJD (2,440,000+)	ΔV	HJD (2,440,000+)	ΔV	HJD (2,440,000+)	ΔV	HJD (2,440,000+)	ΔV
8,783.595......	−0.76	8,783.671......	−1.87	8,784.623......	−1.76	8,795.605......	−0.94
8,783.599......	−0.78	8,783.673......	−1.86	8,784.625......	−1.75	8,795.607......	−0.94
8,783.603......	−0.80	8,783.675......	−1.86	8,784.628......	−1.73	8,795.609......	−0.94
8,783.606......	−0.84	8,783.676......	−1.84	8,784.629......	−1.72	8,795.611......	−0.94
8,783.609......	−0.88	8,783.683......	−1.80	8,784.631......	−1.70	8,800.605......	−1.55
8,783.612......	−0.93	8,783.685......	−1.78	8,784.633......	−1.68	8,800.608......	−1.54
8,783.614......	−0.98	8,783.687......	−1.77	8,784.635......	−1.68	8,800.612......	−1.54
8,783.619......	−1.08	8,783.689......	−1.75	8,784.637......	−1.65	8,800.615......	−1.53
8,783.622......	−1.14	8,783.691......	−1.74	8,784.640......	−1.66	8,800.623......	−1.53
8,783.626......	−1.22	8,783.694......	−1.73	8,784.642......	−1.62	8,800.626......	−1.52
8,783.629......	−1.30	8,783.696......	−1.72	8,784.644......	−1.60	8,800.628......	−1.53
8,783.633......	−1.39	8,783.700......	−1.67	8,784.646......	−1.59	8,800.631......	−1.52
8,783.636......	−1.42	8,783.702......	−1.67	8,784.648......	−1.57	8,800.633......	−1.50
8,783.638......	−1.47	8,783.704......	−1.65	8,784.650......	−1.57	8,800.635......	−1.50
8,783.640......	−1.51	8,783.707......	−1.63	8,784.653......	−1.54	8,801.603......	−1.44
8,783.641......	−1.53	8,783.710......	−1.61	8,784.655......	−1.52	8,801.607......	−1.43
8,783.644......	−1.59	8,783.712......	−1.59	8,784.657......	−1.52	8,801.610......	−1.40
8,783.646......	−1.63	8,784.596......	−1.86	8,784.659......	−1.50	8,801.613......	−1.40
8,783.649......	−1.68	8,784.598......	−1.88	8,784.661......	−1.49	8,801.616......	−1.38
8,783.650......	−1.72	8,784.600......	−1.88	8,784.663......	−1.47	8,801.619......	−1.37
8,783.653......	−1.75	8,784.602......	−1.89	8,784.665......	−1.44	8,801.622......	−1.38
8,783.655......	−1.79	8,784.604......	−1.88	8,784.667......	−1.44	8,801.624......	−1.36
8,783.656......	−1.81	8,784.606......	−1.87	8,784.669......	−1.43	8,801.627......	−1.33
8,783.657......	−1.84	8,784.608......	−1.87	8,795.589......	−0.94	8,801.630......	−1.32
8,783.660......	−1.85	8,784.610......	−1.86	8,795.591......	−0.96	8,802.610......	−1.15
8,783.661......	−1.86	8,784.613......	−1.84	8,795.594......	−0.96	8,802.612......	−1.15
8,783.663......	−1.87	8,784.615......	−1.82	8,795.596......	−0.96	8,802.615......	−1.13
8,783.665......	−1.89	8,784.617......	−1.80	8,795.598......	−0.94	8,802.617......	−1.13
8,783.666......	−1.88	8,784.619......	−1.79	8,795.600......	−0.94	8,802.619......	−1.11
8,783.669......	−1.87	8,784.621......	−1.79	8,795.602......	−0.93	8,802.622......	−1.16

NOTE.—Table 1 is presented in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

shown in Figures 2 and 3, respectively. In phasing these light curves, we used equation (1) below, with the primary period of 0.46998 days.

3. ANALYSIS

Inspection of the light curves shown in Figures 1, 2, and 3 indicates that AR Her exhibited a strong Blazhko effect during 1992 and 1994–1995, with amplitude changes of several tenths of a magnitude and changes in the phase of maximum of a few hundredths of a day.

A discrete Fourier transform (DFT) was initially constructed for the entire 1992–1995 data set. The primary 0.47 day period produced by far the strongest peak in the transform. The gap between the 1992 and 1994 observations resulted in numerous closely spaced aliases for many secondary peaks in the DFT results, but a solution was nonetheless possible for the strong primary pulsation. The

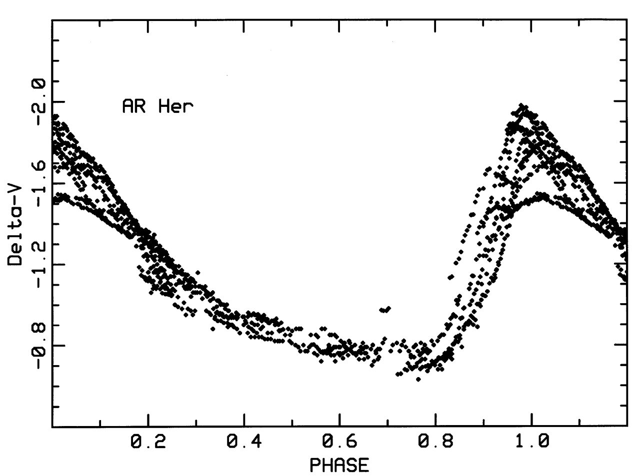

FIG. 1.—Light curve of AR Her, 1992–1995

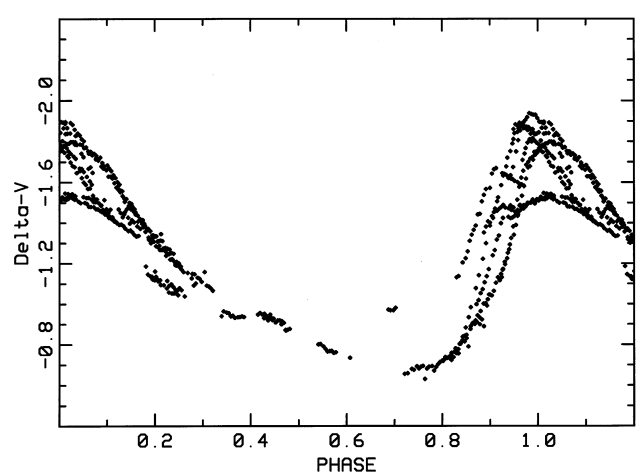

FIG. 2.—Light curve of AR Her, 1992

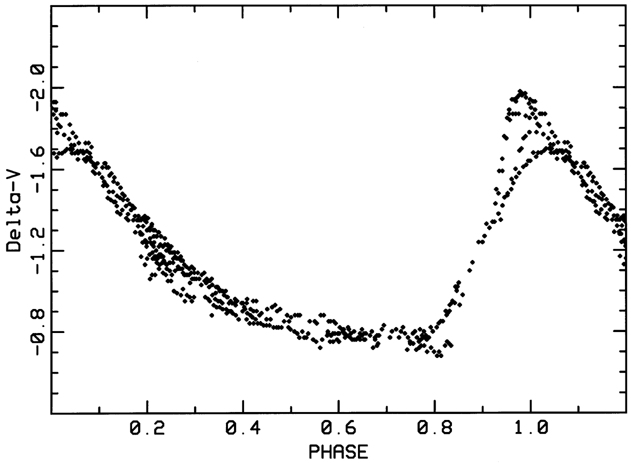

FIG. 3.—Light curve of AR Her, 1995

resultant ephemeris for the mean time of maximum light is

$$\text{Max} = \text{HJD } 2{,}448{,}783.680 \pm 0.003 + (0.469980 \pm 0.000003)E \,, \quad (1)$$

where E is the elapsed cycle number.

This period is slightly shorter than Klepikova's (1957) value of 0.4700234 days for the 1905–1954 interval but is close to that found by Lange (1969) for 1954–1967, 0.469975 days. It is slightly shorter than the 0.470007 day period found by M. E. Baldwin (1992, private communication) from visual observations made between 1985 and 1990. It is likely that these differences signify small but real changes in the primary pulsation period of AR Her.

Figure 4 is a plot of the observed phase of maximum, relative to that predicted by equation (1), versus the brightness of maximum. The point at phase -0.076 and magnitude -1.66 is derived from the combined observations of two consecutive days, JD 2,448,854 and 2,448,855, rather than from a single observed maximum. These show a remarkable short-lived shift to very early maxima during part of the Blazhko cycle. This shift in the phase of maximum is similar to, but is slightly more extreme than, the shifts indicated in similar diagrams for maxima observed in 1937, 1956, and 1958 (Fig. 12, Almar 1961). M. E. Baldwin's (1992, private communication) analysis of AAVSO visual observations of AR Her shows similar very early maxima at the same relative Blazhko phase during 1985–1990. Several data points near phase 0.7 in Figure 2 appear unusually bright. We can find no fault with the observations responsible for these points, which were obtained on JD 2,448,853. The unusual brightness of the observations may be connected with the early maxima on the following two nights.

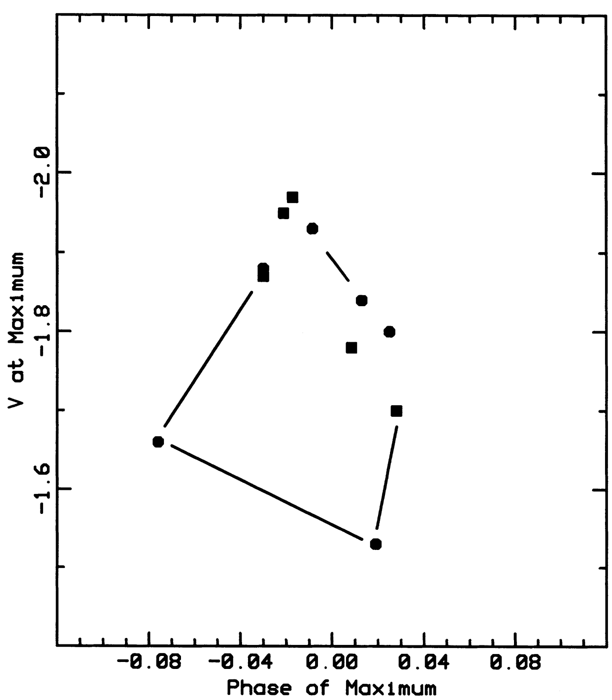

FIG. 4.—Maximum magnitude vs. phase of maximum light. Circles represent 1992 observations. Squares represent observations in 1994–1995. The flow during the progression of the Blazhko cycle is counterclockwise, as indicated by the lines.

M. E. Baldwin (1992, private communication) suggested that, between 1985 and 1990, the Blazhko period of AR Her was near 31.8 days, rather than the 31.55 day period seen earlier (Almar 1961). Because of the gap in our observational coverage, it is difficult to verify this from the 1992–1995 observations. However, such a change seems possible given that the repetition of points in the phase versus magnitude plot of Figure 4 is improved with a 31.77 day period compared with a 31.55 day period.

Because of the aliasing problem created by the gap between the 1992 and 1994 observations, in the remainder of this paper we will (with one exception) model separately the light curves of AR Her in 1992 and 1995. A sinusoidal amplitude modulation of a single oscillation will produce symmetric frequency triplets spaced by the modulation frequency. The symmetry of the triplet sidelobes may be broken by a periodic modulation more complex than a simple sinusoid (Kurtz 1982). The existence of these frequency triplets in the DFT can be indicative of the presence of the Blazhko effect (Smith et al. 1994; Kovács 1995; Alcock et al. 1998).

The initial DFT of the 1992 and 1995 data sets in each case produced a rather complicated pattern of peaks reflecting the time windowing of the observations. The data sets were first prewhitened to remove the primary frequency and its harmonics. In the DFT of the prewhitened data sets, a search was conducted for additional peaks, in particular peaks that might correspond to the frequencies $f_0 \pm f_B$ and $2f_0 \pm f_B$ (where f_0 is the frequency of the primary pusation and f_B the frequency of an approximately 31.6 day Blazhko period). Significant peaks for the $+f_B$ components were readily identified in both the 1992 and 1995 data sets. Weaker peaks associated with the $-f_B$ frequency components were identified in the 1995 data set and, with less clarity, in the 1992 data set.

Discovery in the DFT of frequency triplets spaced according to the Blazhko frequency provides a rationale for modeling the 1992 and 1995 light curves with a more extensive set of frequency triplets. The light curve was modeled by components of frequency

$$\nu = (kf_0 + jf_B) \,, \quad (2)$$

where k is an integer $(1, 2, 3, \ldots)$, $j = -1$, 0, or 1, f_0 is the primary frequency, and f_B is the Blazhko frequency.

As an analysis tool, we used the CLEANest program (Foster 1995), a Fourier analysis tool that derives from the CLEAN routine (Roberts, Lehar, & Dreher 1987). As a test, the CLEANest routine and a least-squares fitting package were used to model the light curve of AH Cam, using the data from Smith et al. (1994). In this case, the fit to the CLEANest solution replicated the least-squares solutions of Smith et al. to within the expected uncertainties of amplitude and phase.

Two models were produced for each data set. In the first, a model was derived with the frequencies kf_0 and $kf_0 \pm f_B$ held fixed. In the second, the frequencies kf_0 and $kf_0 \pm f_B$ were used as starting points for the CLEANest program, but the program was allowed to search the frequency space near the starting values to find the best model. The advantage of the variable-frequency models is that they may find breaks in the symmetry of the frequency triplets that are real. The danger in the variable-frequency solutions is that CLEANest might misinterpret a local maximum due to noise as the best frequency peak. The locked frequency solutions have the advantage of being more directly comparable with prior results and with certain models of the Blazhko effect. Following Alcock et al. (1998), the initial times of the solutions were adjusted to provide equal phases for the $f_0 + f_B$ and $f_0 - f_B$ frequencies. As Alcock et al. noted, for pure amplitude modulation, the phase of f_0 will equal the phases of $f_0 + f_B$ and $f_0 - f_B$. As Tables 2 and 3 indicate, that is not the case for AR Her. This result is expected since Figures 1, 2, and 3 indicate that the Blazhko modulation of AR Her is not a pure amplitude modulation.

The locked frequency terms for models including components of $k = 1$–6 are presented in Tables 2 and 3. The variable-frequency models are listed in Tables 4 and 5. In these models, the Blazhko period of AR Her was taken to be 31.6 days. The solutions are only slightly changed if a Blazhko period of 31.8 days is adopted instead. We note that Borkowski (1980) suggested that it might be more appropriate to model the light curve in intensity rather than magnitude units. To check on whether modeling in intensity units yielded significantly better results, we modeled the data for intensities as well as magnitudes. However, the solutions for intensities did not fit the data significantly better (or worse) than those for magnitudes. The intensity solutions were similar in their essentials to the magnitude solutions, and only the latter will be discussed further.

Both fixed- and variable-frequency models did a reasonably good job of describing the observations. In the case of the locked-frequency models, the standard deviations of the residuals were 0.030 mag for both the 1992 and 1995 data sets. For the variable-frequency models, the standard deviations of the residuals were 0.025 and 0.026 mag for the 1992 and 1995 data sets, respectively. The standard deviations of the residuals are close to, but slightly larger than, the standard deviations expected from observational error alone. Small, but systematic, deviations from the observed magnitudes are evident in the model light curves on certain nights in each data set, indicating that the fits are not quite as good as the observations would allow.

Date-compensated discrete Fourier transforms (DCDFTs) were carried out on the residuals left by the fits

TABLE 2

MODEL OF 1992 DATA: LOCKED FREQUENCIES

Frequency (day^{-1})	Amplitude (mag)	Phase (cycles)	Note
2.1277......	0.391 ± 0.004	0.671 ± 0.011	f_0
2.1594......	0.090 ± 0.004	0.962 ± 0.021	$f_0 + f_B$
2.0960......	0.070 ± 0.004	0.962 ± 0.023	$f_0 - f_B$
4.2553......	0.175 ± 0.004	0.428 ± 0.020	$2f_0$
4.2869......	0.111 ± 0.004	0.636 ± 0.029	$2f_0 + f_B$
4.2236......	0.070 ± 0.004	0.003 ± 0.030	$2f_0 - f_B$
6.3832......	0.032 ± 0.004	0.222 ± 0.031	$3f_0$
6.4148......	0.021 ± 0.004	0.395 ± 0.031	$3f_0 + f_B$
6.3516......	0.015 ± 0.004	0.310 ± 0.032	$3f_0 - f_B$
8.5110......	0.052 ± 0.004	0.009 ± 0.039	$4f_0$
8.5426......	0.053 ± 0.004	0.204 ± 0.039	$4f_0 + f_B$
8.4794......	0.012 ± 0.004	0.646 ± 0.052	$4f_0 - f_B$
10.638......	0.028 ± 0.004	0.710 ± 0.040	$5f_0$
10.670......	0.017 ± 0.004	0.986 ± 0.059	$5f_0 + f_B$
10.607......	0.007 ± 0.004	0.668 ± 0.083	$5f_0 - f_B$
12.766......	0.029 ± 0.004	0.954 ± 0.044	$6f_0$
12.798......	0.013 ± 0.004	0.962 ± 0.046	$6f_0 + f_B$
12.734......	0.022 ± 0.004	0.935 ± 0.040	$6f_0 - f_B$

TABLE 3

MODEL OF 1995 DATA: LOCKED FREQUENCIES

Frequency (day^{-1})	Amplitude (mag)	Phase (cycles)	Note
2.1277......	0.391 ± 0.004	0.183 ± 0.009	f_0
2.1594......	0.089 ± 0.004	0.577 ± 0.018	$f_0 + f_B$
2.0960......	0.046 ± 0.004	0.577 ± 0.020	$f_0 - f_B$
4.2553......	0.164 ± 0.004	0.474 ± 0.017	$2f_0$
4.2869......	0.050 ± 0.004	0.723 ± 0.029	$2f_0 + f_B$
4.2236......	0.020 ± 0.004	0.681 ± 0.032	$2f_0 - f_B$
6.3832......	0.073 ± 0.004	0.851 ± 0.026	$3f_0$
6.4148......	0.080 ± 0.004	0.087 ± 0.030	$3f_0 + f_B$
6.3516......	0.045 ± 0.004	0.186 ± 0.033	$3f_0 - f_B$
8.5110......	0.031 ± 0.004	0.019 ± 0.037	$4f_0$
8.5426......	0.024 ± 0.004	0.614 ± 0.048	$4f_0 + f_B$
8.4794......	0.042 ± 0.004	0.778 ± 0.051	$4f_0 - f_B$
10.638......	0.020 ± 0.004	0.173 ± 0.045	$5f_0$
10.670......	0.022 ± 0.004	0.787 ± 0.061	$5f_0 + f_B$
10.607......	0.008 ± 0.004	0.229 ± 0.085	$5f_0 - f_B$
12.766......	0.009 ± 0.004	0.121 ± 0.038	$6f_0$
12.798......	0.027 ± 0.004	0.112 ± 0.046	$6f_0 + f_B$
12.734......	0.028 ± 0.004	0.203 ± 0.044	$6f_0 - f_B$

TABLE 4

MODEL OF 1992 DATA: VARIABLE FREQUENCIES

Frequency (day^{-1})	Amplitude (mag)	Phase (cycles)	Note
2.1272......	0.420 ± 0.004	0.427 ± 0.007	f_0
2.1584......	0.083 ± 0.004	0.720 ± 0.019	$f_0 + f_B$
2.0969......	0.065 ± 0.004	0.720 ± 0.020	$f_0 - f_B$
4.2571......	0.138 ± 0.004	0.008 ± 0.017	$2f_0$
4.2882......	0.100 ± 0.004	0.329 ± 0.031	$2f_0 + f_B$
4.2265......	0.040 ± 0.004	0.374 ± 0.033	$2f_0 - f_B$
6.3816......	0.091 ± 0.004	0.552 ± 0.028	$3f_0$
6.4144......	0.070 ± 0.004	0.680 ± 0.031	$3f_0 + f_B$
6.3371......	0.035 ± 0.004	0.722 ± 0.031	$3f_0 - f_B$
8.5179......	0.014 ± 0.004	0.487 ± 0.039	$4f_0$
8.5397......	0.061 ± 0.004	0.487 ± 0.038	$4f_0 + f_B$
8.4818......	0.029 ± 0.004	0.653 ± 0.044	$4f_0 - f_B$
10.638......	0.041 ± 0.004	0.653 ± 0.037	$5f_0$
10.672......	0.042 ± 0.004	0.902 ± 0.045	$5f_0 + f_B$
10.582......	0.026 ± 0.004	0.826 ± 0.050	$5f_0 - f_B$
12.771......	0.025 ± 0.004	0.623 ± 0.043	$6f_0$
12.804......	0.026 ± 0.004	0.712 ± 0.049	$6f_0 + f_B$
12.739......	0.017 ± 0.004	0.475 ± 0.052	$6f_0 - f_B$

TABLE 5

MODEL OF 1995 DATA: VARIABLE FREQUENCIES

Frequency (day^{-1})	Amplitude (mag)	Phase (cycles)	Note
2.1276......	0.382 ± 0.004	0.224 ± 0.009	f_0
2.1592......	0.102 ± 0.004	0.637 ± 0.019	$f_0 + f_B$
2.0964......	0.056 ± 0.004	0.637 ± 0.025	$f_0 - f_B$
4.2553......	0.177 ± 0.004	0.555 ± 0.016	$2f_0$
4.2869......	0.034 ± 0.004	0.774 ± 0.034	$2f_0 + f_B$
4.2219......	0.020 ± 0.004	0.726 ± 0.033	$2f_0 - f_B$
6.3828......	0.075 ± 0.004	0.973 ± 0.028	$3f_0$
6.4144......	0.068 ± 0.004	0.183 ± 0.033	$3f_0 + f_B$
6.3512......	0.043 ± 0.004	0.271 ± 0.035	$3f_0 - f_B$
8.5106......	0.036 ± 0.004	0.181 ± 0.042	$4f_0$
8.5470......	0.027 ± 0.004	0.661 ± 0.051	$4f_0 + f_B$
8.4818......	0.050 ± 0.004	0.957 ± 0.044	$4f_0 - f_B$
10.661......	0.018 ± 0.004	0.877 ± 0.055	$5f_0$
10.670......	0.048 ± 0.004	0.994 ± 0.060	$5f_0 + f_B$
10.595......	0.018 ± 0.004	0.451 ± 0.073	$5f_0 - f_B$
12.781......	0.029 ± 0.004	0.481 ± 0.064	$6f_0$
12.798......	0.019 ± 0.004	0.474 ± 0.069	$6f_0 + f_B$
12.734......	0.025 ± 0.004	0.505 ± 0.065	$6f_0 - f_B$

described in Tables 2, 3, 4, and 5, but no clear evidence was detected of additional significant periodic terms. Plots of amplitude versus frequency are shown in Figures 5 and 6 for the DCDFTs of the residuals from the variable-frequency fits to the 1992 and 1995 data, respectively. In the DCDFT for the 1995 data, there is a hint of a periodicity, but of such low amplitude that its inclusion or exclusion in a fit makes little difference in the size of the residuals.

In Smith et al.'s (1994) analysis of observations of AH Cam, it was found that inclusion of a frequency equal to the Blazhko frequency, f_B, improved the fit to the 1991–1992 data. In the case of AR Her, models including the Blazhko frequency produced no significant improvement in the fit to the observations. We also did not find evidence for the 90.8 day tertiary period reported by Almar (1961), although such a long period, if of small amplitude, may be difficult to detect in the observations of a single year.

Despite the difficulties with aliasing alluded to above, a fit was attempted for the entire 1992–1995 data set, using fixed frequencies and a Blazhko period of 31.77 days. With terms for $k = 1$–6, the fit was not as successful as the modeling of data for individual years. The standard deviation of the residuals to the fit was ± 0.046 mag. This is significantly larger than the residuals found for the individual models of the 1992 and 1995 data sets, and the instances of significant systematic deviations of the predicted magnitudes from those observed were more pronounced in the global solution. The Blazhko effect in AR Her, or indeed its primary pulsation, may have changed amplitude, frequency, and/or phase slightly between 1992 and 1995, creating one source of difficulty in the modeling.

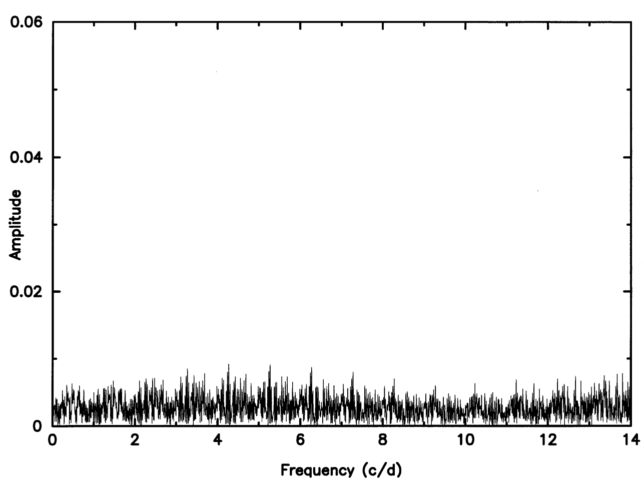

FIG. 5.—Date-compensated discrete Fourier transform of the residuals from the 1992 variable-frequency solution.

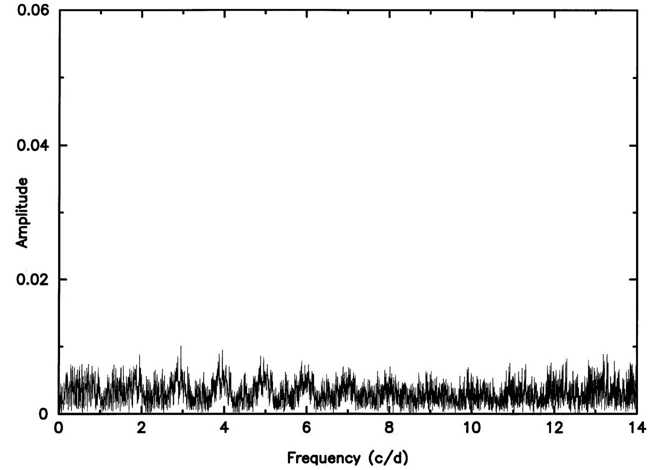

FIG. 6.—Same as Fig. 5, but from the 1995 variable-frequency solution

4. COMPARISON WITH PRIOR RESULTS

AR Her is one of the few RR Lyrae stars showing the Blazhko effect to have had its light curve modeled previously. The approach most similar to our own was that of Borkowski (1980), who modeled 1363 visual observations of AR Her obtained in 1944 (Tsessevich & Ustinov 1953). The form of Borkowski's model is close to that used here. Borkowski held the frequency terms fixed, as in our locked models, in obtaining a least-squares fit with terms extending to $k = 7$ in our notation. He also included a component with the Blazhko frequency, f_B, though the amplitude of that component was relatively small, about 0.04 mag.

Borkowski's results are similar to the models of our 1992 observations in having an amplitude for the $2f_0 + f_B$ component stronger than for the $f_0 + f_B$ component. It is noteworthy, however, that the 1992 data set and the data set used by Borkowski are both biased by having more observations near maximum than minimum light. In Borkowski's model, the amplitudes of the $+f_B$ components are larger than those of the $-f_B$ components. The same is generally but not invariably true for the models of the more recent observations: The amplitudes of the $+f_B$ components are usually, but not always, larger than the $-f_B$ components in the fits to the 1992 and 1995 observations.

Almar (1961) used photographic observations of AR Her made in 1935–1939 and 1955–1957 to construct "mean light surfaces" showing the light-curve changes of AR Her as a function of the primary and Blazhko cycles. He expanded the mean light surfaces into trigonometric polynomials. In doing this, as Borkowski noted, Almar essentially assumed that the light curve of AR Her could be constructed from sinusoidal components of frequency $kf_0 + jf_B$, where $j = 0$, 1 or -1, that is, a model similar in form to that adopted here. In these models, the $2f_0 + f_B$ components have amplitudes distinctly smaller than the $f_0 + f_B$ terms, as was also the case with our models of the 1995 observations. In Almar's polynomials, we again find that the amplitudes of $+f_B$ components are larger than those of the $-f_B$ components.

5. DISCUSSION

In this section we address two questions: (1) Is AR Her likely to be a binary star, consisting of RR*ab* and RR*c* components, as suggested by Kinman & Caretta (1992), and (2) how do the results of the AR Her analysis compare with predictions of models of the Blazhko effect?

5.1. *Is AR Her Composed of Two RR Lyrae Stars?*

Kinman & Caretta (1992) were led to suggest that AR Her might be a binary composed of an RR*ab* and an RR*c* star for two reasons. First was the evidence that AR Her had an unusually high surface temperature for an RR*ab* star: its unusually blue color and strong hydrogen lines near minimum light. Second was Borkowski's (1980) finding of a large amplitude for the 0.23 day, $2f_0 + f_B$ component in his analysis of its light curve. Kinman & Caretta pointed out that, if AR Her were an unresolved binary composed of an RR*ab* star of 0.47 day period and an RR*c* star of period 0.23 days, then the unusual temperature of AR Her and its Blazhko effect might both be explained.

There are, however, strong arguments against this possibility. The detailed properties of the light curve of AR Her are not in accord with a linear combination of the light curves of a 0.47 day RR*ab* and a 0.23 day RR*c* star. In a Fourier analysis of the intensity data, a linear superposition of RR*ab* and RR*c* type light curves would be expected to produce for each star significant amplitudes for the primary frequency and its lower harmonics. For the RR*c* star, the largest amplitude would be expected to be that of the 0.23 day primary period, represented by the $2f_0 + f_B$ component. However, the amplitude of the $2f_0 + f_B$ component is not particularly large compared with other $+f_B$ components in the fits to the 1995 data (either in intensity or magnitude units). That component is relatively strong in the models of the 1992 data, but even there it is only slightly stronger than the $f_0 + f_B$ component. Nor is the $2f_0 + f_B$ component particularly strong in Almar's (1961) analyses of the light surfaces of AR Her in 1937 and 1956. The problem with reproducing the observed AR Her light curve by combining the light curve of an RR*ab* variable with that of even a relatively large amplitude RR*c* star is illustrated by considering the rising branch. At phases shortly before the mean phase of maximum light, the brightness of AR Her can differ by more than 0.6 mag during different parts of the Blazhko cycle (see Fig. 1). No comparably large brightness differences are observed on the declining branch of the AR Her light curve, even as the decline approaches minimum, although such would be expected if the Blazhko effect were the result of the combination of RR*ab* and RR*c* light curves. This is true not only for our data set, but also for the mean light curves in Almar, which more completely sample certain phases of the Blazhko cycle.

We conclude, therefore, that the Blazhko effect in AR Her is not the result of a linear combination of RR*ab* and RR*c* light curves. The possibility remains that the bluer color of AR Her at minimum, if real when averaged over the full Blazhko cycle, is the consequence of AR Her being an unresolved binary in which one component is a blue, nonvariable star. We note, however, that the maximum amplitude of AR Her, about 1.3 mag in V, is not unusually small for a star of its period (see Preston 1959). Thus, any nonvariable component cannot be so bright in V (or in B; see the light curves in Almar 1961) as to substantially reduce the amplitude of AR Her. High-resolution spectroscopic observations, or observations in the ultraviolet part of the spectrum, might shed light upon this question.

5.2. *AR Her and the Nature of the Blazhko Effect*

Recent explanations for the Blazhko effect have centered on two alternatives. In the first, the Blazhko modulation is regarded as induced in rotating RR Lyrae stars, perhaps stars that have strong magnetic fields (Detre & Szeidl 1973; Cousens 1983; Kovács 1995). In the second, the Blazhko effect is considered to be produced by interaction of different pulsation modes (Borkowski 1980; Moskalik 1986; Cox 1993; Kovács 1995).

Borkowski (1980) suggested that the Blazhko effect in AR Her arose though the nonlinear superposition of the fundamental radial mode and a mode with frequency $2f_0 + f_B$, which he identified as either the second or third overtone. Borkowski was partly driven to this conclusion by the relative prominence of the $2f_0 + f_B$ component and the $+f_B$ components in general in his least-squares fit to the Tsessevich & Ustinov 1953 observations. Moskalik (1986) similarly suggested that resonant coupling between the fundamental and third-overtone radial modes might produce the Blazhko effect. This proposal has, however, run into difficulties because theoretical pulsational models have not supported resonant excitation of purely radial modes, leading recent investigations to focus on other explanations (Kovács 1995). We note as well that the $2f_0 + f_B$ component is not as prominent in our analyses as in Borkowski's, particularly as regards the 1995 observations.

Kovács (1995), while pointing out that photometric observations alone may prove inadequate to verify explanations of the Blazhko effect, did note that certain simple tests were possible with photometric data. He adapted the oblique rotator-pulsator model originally developed for rapidly oscillating Ap stars (Kurtz 1982) to the case of the Blazhko variables. In this model, a single nonradial mode of $m = 0$ is coupled with the radial fundamental mode. The Blazhko period in this model is the same as the rotational period of the star. Kovács predicted that the parameter combination

$$R = A_+(k)/A_-(k) \quad (3)$$

should be independent of the component under consideration, where $A_+(k)$ and $A_-(k)$ are the amplitudes of the $kf_0 + f_B$ and the $kf_0 - f_B$ components. The difference in the phases, $\Delta\phi$, between the $+f_B$ and $-f_B$ components is also expected to be constant under this model.

Because of Kovács's doubts as to whether photometry alone can test adequately models of the Blazhko effect and because of doubts as to the ability of this simple rotator-pulsator model to predict the R-ratios and $\Delta\phi$ (D. W. Kurtz 1999, private communication), we list values of R and $\Delta\phi$ more as possibly interesting diagnostics than as tests of the oblique rotator-pulsator model. Values of R and $\Delta\phi$ for our fixed-frequency models and for Borkowski's (1980) model of the Tsessevich & Ustinov 1953 observations are shown in Tables 6 and 7.

The results of Borkowski's analysis are consistent with the predictions of the Kovács simple oblique rotator-pulsator model. However, the clustering of the R-values around 2.0 must be fortuitously tight given the large uncertainties in many of the ratios. The results for the 1992 data set are not in such good accord with the predictions, although R is reasonably constant for the $k = 1$–3 components. Results for the $k = 1$–3 components of the modeling of the 1995 data set are in reasonable accord with the predictions, but that is not true for the $k = 4$–6 components. In any case, this very simple model may be, as we have noted, inadequate in various aspects, and its predictions may not hold for more general applications of the oblique rotator-pulsator model.

Kovács (1995) noted two other problems with the oblique rotator-pulsator model for the Blazhko effect. In these models, the amplitude of the Blazhko effect should depend upon the aspect angle at which the star is observed. Kovács noted that all known Blazhko stars have modulation amplitudes of 0.3–0.7 mag (V or B) (Szeidl 1988). Although it is true that the modulation amplitude of a Blazhko star can change on a time span of years and although there may be bias against the discovery of Blazhko effects of very small amplitude, the limited range of modulation of the Blazhko effect may be significant. Kovács also argued that the Blazhko stars fit the standard period-amplitude relationship when they are in their high-amplitude state (Szeidl 1988), whereas the model in which the Blazhko pulsation is coupled to oblique rotation predicts that the high- and low-amplitude states in the Blazhko cycle should be symmetric around the relation for non-Blazhko stars. We note, however, that not all versions of the oblique rotator-pulsator model may require this symmetry (Alcock et al. 1998).

Alcock et al. introduced an important new observational approach to the problem of the Blazhko effect, using observations obtained in the MACHO survey to analyze the Blazhko effect in three RR Lyrae stars. Alcock et al. were able to describe the Blazhko effect in these three stars as showing nearly pure amplitude modulation of a single pulsation mode. This they found to be consistent with a version of the oblique rotator-pulsator model developed by Takata & Shibahashi (1999).

In the Takata & Shibahashi model, an oblique, dipolar magnetic field modulates the amplitude of the fundamental radial mode. The observed amplitude of the fundamental mode changes as the star rotates, and the Blazhko period is equal to the rotation period. However, as Alcock et al. noted, not all Blazhko-effect stars show pure amplitude modulation. Blazhko stars such as AR Her show a more complicated pattern of light-curve changes. It is clear, however, that the wealth of data available from the MACHO survey is going to make a large contribution to the solution of the Blazhko problem.

There may, however, be a neglected problem for models of the Blazhko effect that require the Blazhko period be always equal to the rotation period of the Blazhko star. For some Blazhko stars there are reports in the literature that, when the primary pulsation period of the star changes, its Blazhko period also changes—and in the opposite sense. The possible recent increase in the Blazhko period of AR Her from about 31.6 to 31.8 days may be associated with a coincident small decrease in the primary period. However, the effect is better seen in stars that have undergone larger changes in primary period than has AR Her. RW Dra (Tsessevich 1966) is a possible exemplar of this phenomenon. XZ Cyg is another. XZ Cyg underwent a relatively large decrease in its primary period around JD 2,438,700. The fundamental period fell, possibly in several steps, from 0.466579 to about 0.46648 days (Taylor 1979; Bezdenezhnyi 1988). At the same time, the Blazhko period of XZ Cyg appears to have increased from 57.4 ± 0.1 to 58.4 ± 0.2 days (Baldwin 1973; Smith 1975; Kunchev 1975; Taylor 1979; Pop 1975).

The change in primary period, though relatively large for an RR Lyrae star, corresponds to a very small change in stellar density according to the pulsation equation $P\sqrt{\rho} = Q$ (assuming a constant value of the pulsation constant Q): $\Delta\rho = 0.015\%$. Alternatively, a change in the structure of the star might result in a small change in the pulsation constant Q. It would seem unlikely that such a small change in density or Q would be associated with an almost 2% change in the rotation period of the star. Such a relatively large change in the rotation period of XZ Cyg would require a drastic redistribution of angular momentum, which seems very unlikely. If the rotation period did not increase from 57.4 to 58.4 days, then the Blazhko cycle is not in this case equal to the rotation period.

Ledoux (1951; see also Kurtz 1982) noted that rotationally perturbed nonradial m-modes can produce frequency triplets in which the frequency spacing is not directly equal to the rotation period, the difference depending upon the

TABLE 6

Light-Curve Diagnostics

k	R	$\Delta\phi$ (cycles)
1992:		
1......	1.2 ± 0.2	$+0.00 \pm 0.03$
2......	1.6 ± 0.2	$+0.63 \pm 0.04$
3......	1.4 ± 0.2	$+0.08 \pm 0.04$
4......	4.4 ± 0.4	-0.44 ± 0.06
5......	2.4 ± 1.6	$+0.32 \pm 0.10$
6......	0.6 ± 0.2	$+0.03 \pm 0.06$
1995:		
1......	1.9 ± 0.2	$+0.00 \pm 0.03$
2......	2.5 ± 0.2	$+0.04 \pm 0.04$
3......	1.8 ± 0.2	-0.10 ± 0.04
4......	0.6 ± 0.2	-0.16 ± 0.07
5......	2.8 ± 1.4	$+0.56 \pm 0.10$
6......	1.0 ± 0.2	-0.09 ± 0.06

TABLE 7

Light-Curve Diagnostics: Borkowski Fit

k	R	$\Delta\phi$ (cycles)
1......	2.0 ± 0.4	0.62 ± 0.03
2......	2.4 ± 0.4	0.64 ± 0.04
3......	2.9 ± 1.1	0.58 ± 0.06
4......	1.8 ± 0.6	0.55 ± 0.06
5......	2.1 ± 1.1	0.56 ± 0.08
6......	2.0 ± 1.8	0.54 ± 0.14
7......	2.2 ± 1.5	0.59 ± 0.11

pulsation mode and the structure of the star. There seems, however, no way to tie this possibility to the period changes of the Blazhko effect observed in RR Lyrae stars.

We may also note that, if the Blazhko period is interpreted as some form of beat period, then the observations of period change indicate that, when the primary period changes, the period against which it is beating cannot change in equal proportion—or else the observed changes in the Blazhko period would not be reproduced. The recent finding of Paparo et al. (1998), that the fundamental mode of a double-mode RR Lyrae variable can increase while its first-overtone mode decreases, indicates that period changes in RR Lyrae stars may be more complicated than hitherto believed and may be relevant to the case of the Blazhko stars.

6. SUMMARY

AR Her continued to show a large Blazhko effect in 1992 and 1995, with changes in maximum magnitude and phase of maximum light similar to those seen in earlier years. The primary period of AR Her during 1992–1995 was 0.46998 days. The light-curve modulation of AR Her is not likely to be produced by the superposition of the light curves of an RR*ab* and an RR*c* star. The light curves of AR Her in 1992 and 1995 could be reasonably well, but not perfectly, described by models superposing components of frequency $kf_0 + jf_B$, where f_0 is the primary frequency, f_B is the Blazhko frequency, $k = 1$–6 and $j = -1, 0, 1$. The phases of the frequency components indicate that AR Her does not show pure amplitude modulation. The coincident changes in primary and Blazhko period reported for RW Dra and XZ Cyg pose challenges for models of the Blazhko effect in which the Blazhko period is identical to the rotation period of the star.

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